PECULIARITIES OF THE RESPONSE OF A LASER INTERFERENCE GRAVITATIONAL ANTENNA TO LOW-FREQUENCY DISTURBANCES

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The extensive use of Fabry–Perot interferometers for measuring small relative displacements in applied [1] and fundamental [2] (for instance, recording of gravitational waves [3]) research calls for study of the possible mechanisms of transformation of low-frequency noise disturbances to high-frequency types of approximately the same frequency as the recorded useful signal. The study acquires considerable interest in view of the development of wide-base laser interference gravitational antennas [4]. At the same time, the high cost, large overall dimensions, and increased complications in manufacture and operation of such installations make it necessary, in the first instance, to analytically define the transformation of low-frequency seismic disturbances to the range of working frequencies ($\gtrsim$1 kHz) and, based on this, to optimize laser interference gravitational antennas for reducing the level of random measuring errors.

Laser interference gravitational antennas being developed now are interferometers built up on Michelson's circuit, each arm of which is assembled as a Fabry–Perot resonator. The resonator mirrors S_1 and S_2 (S_1 is a partially transmitting mirror) are mounted on the test bodies which are weakly connected with the base. Since a Fabry–Perot resonator is a multibeam interferometer, then, radiation pressure on mirror S_2 (and, consequently, on the test body) depends significantly on its possible displacement in the direction of the resonator's optical axis. Such displacements can occur as a result of low-frequency microseismic [5] and a series of other disturbances (thermal noise, geomagnetic pulsations, effect of cosmic rays, et al. [6]). It is of interest to examine the possibility of transformation of these low-frequency disturbances to high-frequency fluctuations of the signal recorded by the interferometer. This transformation can be caused, in particular, by nonlinearity of the equation of motion of the test body in the radiation pressure field.

We examine a Fabry–Perot optical resonator, the radiation source for which is a laser of intensity I_0, creating amplitude E_0 of the light wave at the input to the resonator. We assume that mirror S_1 is characterized by energy reflection coefficient R_1 and transmission factor η_1 and, mirror S_2, by R_2 and η_2. In this, losses are considered to be small, i.e., $1 - (R_i + \eta_i) \ll 1$, $i = 1, 2$. We assume that one of the test bodies with mirror S_2 mounted on it undergoes an irregular action leading to its displacement from the equilibrium position by amount $x(t)$. Then, assuming that the direction of propagation of the beam coming from the laser is positive, we can write the amplitude of ray n incident on the mirror as

$$E_n=E_0\sqrt{\eta_1}(R_1R_2)^{(n-1)/2}\cos[\omega_c t+\varphi_0+(n-1)\alpha-k(2L(n-1)+x(t)+2\sum_{i=1}^{n-1}x(t-i\Delta t))],$$

where ω_c is cyclic frequency of laser radiation; φ_0 is initial phase,; α is total losses of the phase with one-time rereflection from the resonator mirrors; k is wave number; L is undisturbed length of resonator; $\Delta t \simeq 2L/c$ is time interval between successive rereflections of the ray by mirror S_2; c is velocity of light. Amplitudes for the rays reflected from mirror S_2 and emergent from the resonator are equal to

$$E_n'=\sqrt{R_2}E_n,\quad E_n''=\sqrt{\eta_1 R_2}E_n.$$

The following expressions will respectively characterize the intensity of radiation pressure $F(x, t)$ on mirror S_2 and intensity of radiation $I(x, t)$ emergent from the resonator:

$$F(x,t)=(2I_0/cE_0^2)(1+R_2)\langle E^2\rangle;\quad I(x,t)=(2I_0/E_0^2)R_2\eta_1\langle E^2\rangle, \tag{1}$$

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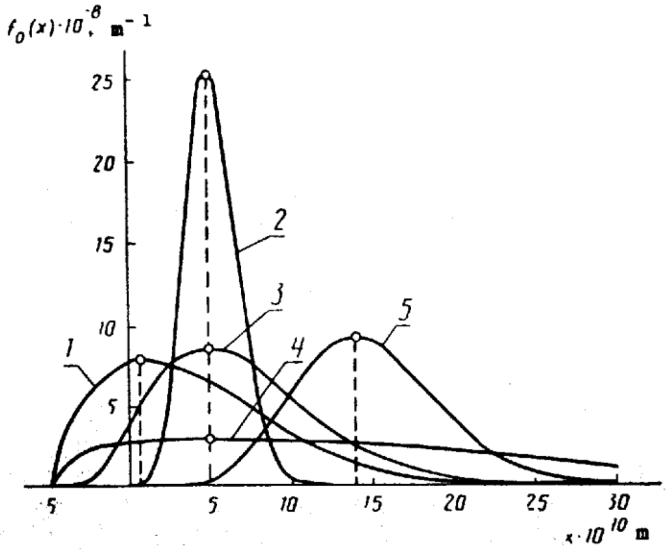

Fig. 1

where $\langle E^2\rangle = \lim_{N\to\infty}\langle(\sum_{n=1}^{N} E_n)^2\rangle$, $\langle\ldots\rangle$ is the averaging operation for one period of the light wave. Considering that average amplitude of oscillations of the test body $\langle A_x\rangle$ is small, i.e., $\langle A_x\rangle \ll L/N_0$ (N_0 is average number of rereflections in the resonator), and assuming that relationship $2kL - \alpha = 2\pi\mu + \varkappa$ (μ is a whole number; $\varkappa$ is phase shift characterizing resonator tuning) takes place for the resonator, we can obtain the following expression:

$$\langle E^2\rangle = (E_0^2/2)\lim_{N\to\infty}\left\langle\left\{\sum_{n=1}^{N}(R_1R_2)^{n-1} + 2\sum_{m=1}^{N-1}\sum_{n=m+1}^{N}(R_1R_2)^{(n+m-1)/2}\cos\left(2k\sum_{i=m}^{n-1}x(t-i\Delta t)+(n-m)\varkappa\right)\right\}\right\rangle. \tag{2}$$

If in case oscillations of the test body are low-frequency type, and their limiting frequency $\nu_0 \ll 1/N_0\Delta t$, then $x(t - i\Delta t)$ in (2) can be replaced by $x(t)$. Accounting for the fact that losses in one cycle of rereflections of the ray in the resonator $\Delta = 1 - \sqrt{R_1R_2}$ are small ($\Delta \ll 1$), expression (2) takes the following simple form after summation:

$$\langle E^2\rangle = (E_0^2\eta_1/2)/(\Delta^2 + 4(1-\Delta)\sin^2(kx(t)+\varkappa/2)). \tag{3}$$

Substitution of (3) in (1) yields the final expressions for the intensity of radiation pressure on the test body $F(x)$ and intensity of radiation emergent from the resonator $I(x)$.

If the resonator is out of adjustment such that $\Delta < kx+\varkappa/2 \ll 1$, and this corresponds to the working point for Fabry–Perot interferometers, then the expression for the intensity of radiation pressure on the test body can be represented as

$$F(x) = (I_0\eta_1/2ck^2)/(x+r)^2, \tag{4}$$

where the notation $r = \varkappa/2k$ is introduced and we take into account that $1 - R_2 \ll 1$.

Substitution in (4) of the parameters $I_0 = 10^2$ W, $k \cong 10^7$ m^{-1}, $\varkappa = 0.01$, and $\eta_1 = 10^{-3}$, characteristic for installations under construction in the California and Massachusets Institutes of Technology [4], yields $F = 10^{-6}$-10^{-5} N. Taking into account its nonlinear dependence on x, such an intensity can substantially change the character of motion of the test body.

The equation of oscillations of the test body in the radiation pressure field can be written as

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x - b/(x+r)^2 = \xi(t), \tag{5}$$

where β is damping coefficient; ω_0 is the natural frequency of oscillations of test body in the absence of radiation pressure; $b = I_0\eta_1/2McK^2$; $\xi(t)$ is coercing force; M is test body mass. If $\xi(t)$ can be approximated by white Gaussian noise with intensity 2D (this can, for example, be microseismic noise), then, the Fokker–Planck equation [7] can be constructed for (5).

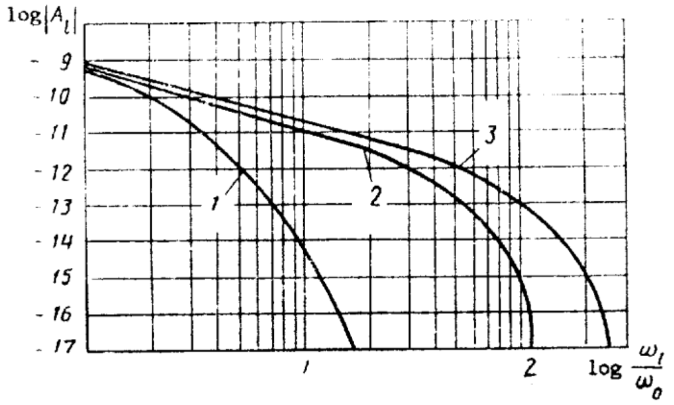

Fig. 2

This equation gives the distribution function f(x, p, t) of fluctuations of coordinate x and pulse $p = M\dot{x}$ of the test body as follows:

$$\frac{\partial f}{\partial t}+(p/M)\frac{\partial f}{\partial x}+[Mb/(x+r)^2-2\beta p\ -M\omega_0^2 x]\frac{\partial f}{\partial p}\ -2\beta f=D\ \frac{\partial^2 f}{\partial p^2}\ . \qquad (6)$$

A stable solution of f(x, p) is the combined probability density of coordinate x and pulse p of the test body. Integration of the distribution function f(x, p) for all possible values of the pulse makes it possible to obtain the expression for the steady distribution of fluctuations of test body with respect to coordinate x:

$$f_0(x)=S\exp[-(M\omega_0^2x^2/2-V(x))/W], \qquad (7)$$

where

$$S^{-1}=\int_{-r}^{\infty}\exp\ [-(M\omega_0^2x^2/2-V(x))/W];$$

$V(x) = -Mb/(x + r)$ is potential energy of radiation pressure field; $W = 2\beta M/D$ is average energy or random oscillations of test body.

The distribution function (7) differs from the Gaussian. This is connected with the nonlinear character of oscillations described by (5) and points to the possibility of occurrence of the high-frequency components of fluctuations of the coordinate of the test body. Graphs of $f_0(x)$ calculated by (7) for an interference gravitational antenna with $M = 10^2$ kg [8] and $\omega_0^2 = 20\ sec^{-2}$ having the following values of energy W of random vibrations of the test body and coefficient b (characterizing the parameters of the laser-resonator system) are given in Fig. 1: 1) $W = 10^{-15}$ J and $b = 10^{-27}$ J·m/kg; 2) $W = 10^{-16}$ J and $b = 10^{-26}$ J·m/kg; 3) $W = 10^{-15}$ J and $b = 10^{-26}$ J·m/kg; 4) $W = 10^{-14}$ J and $b = 10^{-26}$ J·m/kg; 5) $W = 10^{-15}$ J and $b = 10^{-25}$ J·m/kg. From comparison of the graphs, it follows that with increase in radiation pressure, characterized by parameter b, the equilibrium position of the test body is displaced (graphs 1, 3, 5). This leads to variation in the operating point of the resonator. Increase in energy W causes broadening of the distribution function $f_0(x)$ and intensification of distortions of its form in comparison with a Gaussian distribution (graphs 2, 3, and 4).

We make use of the following method to determine the character of transformation of low-frequency oscillations to high-frequency oscillations of its fluctuations. We assume that after a single external pulse action, caused, for instance, by the passage of a seismic wave, the test body has acquired energy W and the dissipation of this energy in time $\tau \simeq 1/\beta$, during which, the oscillations can be assumed to be free. Assuming that the oscillating system is of adequately high quality $\beta \ll \omega_0$, its period corresponding to energy W can be determined by the formula [9].

$$T(W)=\int_{x_2}^{x_1}(\sqrt{2M}dx/\sqrt{W-\Pi_0(x)}),$$

where $\Pi_0(x) = M\omega_0^2x^2/2 - V(x)$, is total potential energy of the test body in the gravitational force and radiation pressure fields; x_1 and x_2 are values of the coordinates at which the potential energy $\Pi_0(x)$ becomes equal to W, i.e., $\Pi_0(x_{1,2}) = W$.

The period of oscillations of the test body as applied to the examined system, takes the form

$$T(W)=\frac{2}{\omega_0}\int_{x_2}^{x_1}(\sqrt{x+r}dx/\sqrt{-x^3-rx^2+a_1x-a_2}). \tag{8}$$

where the following notations are introduced: $a_1 = 2W/M\omega_0^2$; $a_2 = 2(Mb - Wr)/M\omega_0^2$.

Finding the integral in (8) gives

$$T(W)=(2(x_2+r)/\omega_0\sqrt{(x_1+r)(x_2-x_3)}\ \Pi(\pi/2,\rho,q). \tag{9}$$

where x_1, x_2, and x_3 are roots of the cubic equation in the denominator of integral (8) and, here, $x_1 > x_2 > x_3$; $\Pi(\pi/2, \rho, q)$ is the elliptical integral of third order; $\rho = (x_2 - x_1)/(x_1 + r)$; $q = \sqrt{(x_2 - x_1)(x_3 + r)/(x_1 + r)(x_2 - x_1)}$.

Substitution of the values characteristic of the laser—resonator system ($W = 10^{-14}$ J, $b = 10^{-26}$ J·m/kg, $\omega_0 = 4.5$ sec^{-1}, and $r = 5\cdot10^{-10}$ m) in (9) makes it possible to determine the natural oscillations and, hence, even their frequency $\omega = 2\pi/T(W) = 9.6$ sec^{-1} which is two times higher than frequency ω_0.

The free oscillations of the test body in time $\tau < 1/\beta$ after passage of the seismic wave can be described by equation (5) for $\xi(t) = 0$ and in disregarding the term $2\beta\dot{x}$ which accounts for damping. The periodic solution for $x(t)$ of the equation thus obtained is presented in [9] as a Fourier series for the natural frequency ω as follows:

$$x(t)=A_0+\sum_{i=1}^{\infty}A_i\cos i\omega t. \tag{10}$$

Substitution of (10) in the examined equation of free undamped oscillations of the test body gives

$$(A_0+r)^2\omega_0^2A_0-b+\sum_{k=1}^{\infty}\alpha_kA_k\cos k\omega t+\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\beta_kA_iA_k\cos i\omega t\cos k\omega t+$$
$$+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\gamma_kA_iA_jA_k\cos i\omega t\cos j\omega t\cos k\omega t=0, \tag{11}$$

where

$$\alpha_k=(A_0+r)^2\gamma_k+2(A_0+r)\omega_0^2A_0;\ \ \beta_k=2(A_0+r)\gamma_k+\omega_0^2A_0;$$
$$\gamma_k=\omega_0^2-k^2\omega^2.$$

Considering that $A_1 < A_0 + r$, $A_{i+1} < A_i$, and $i = \overline{1, n}$, the relationship (11) makes it possible to determine the recurring formula for amplitude A_ℓ higher than the first order after grouping terms having the same frequency $\omega_e = \ell\omega$ and accounting for triviality of terms containing the amplitude A_i for $i > \ell$, i.e.,

$$A_l=-\frac{1}{2}\sum_{i,k=1}^{i+k=l}\beta_{k,l}A_iA_k-\frac{1}{4}\sum_{i,j=1}^{i,j<l}A_iA_j(\gamma_{i+j-l,l}A_{i+j-l}+2\gamma_{l,l}A_{i+j-l})-\frac{1}{4}\sum_{i,j,k=1}^{i+j+k=l}A_iA_jA_k\gamma_{k,l},\ l=\overline{2,n}, \tag{12}$$

where $\beta_{k,\ell} = \beta_k/\alpha_\ell$, $\gamma_{k,\ell} = \gamma_k/\alpha_\ell$, and $\sum_{i,k=1}^{i+k=l}$ denotes summation of all terms with $i \geq 1$ and $k \geq 1$ under the condition that $i + k = \ell$.

Values of A_0 and A_1 which depend on W and b are required to be set for direct calculation of amplitude A_ℓ by (12). The amount of steady displacement can be estimated in the first approximation from the position of the maximum in the graphs in Fig. 1 while the amplitude of the first harmonic A_1 is connected with energy W by the approximate relationship $W \approx Mb/(A_0 + r) + M\omega^2A_1^2/2$.

We analyze the dependence of the amplitude of oscillations on frequency $|A_\ell(\omega_\ell')|$ after setting values of $b = 10^{-26}$ J·m/kg, $W = 10^{-15}$-10^{-14} J, and $r = 5\cdot10^{-10}$ m (here, $A_0 = 5\cdot10^{-10}$ m and $A_1 = (5\text{-}10)\cdot10^{-10}$ m), and considering that the other parameters of the laser—resonator system are approximately the same as for the gravitational antennas under construction.

Graphs of the dependence of $|A_\ell(\omega_\ell)|$ with the following values of A_1 are given in Fig. 2 in logarithmic scale: 1) $A_1 = 0.5(A_0 + r)$; 2) $A_1 = 0.9(A_0 + r)$; 3) $A_1 = 0.95(A_0 + r)$. As it follows from the graphs, the amplitude of the hundredth harmonic A_{100} for the case A_1 = 0.9 (a_0 + r) is in the region of 10^{-15} m, while for A_1 0.95 (A_0 + r) it takes a value of approximately 10^{-13} m. With growth in energy of random low-frequency oscillations of the test body, there is corresponding increase in amplitude of the high-frequency harmonics and it can exceed the proposed level of sensitivity ($\Delta x = 2 \cdot 10^{-15}$ m) in the working frequency range of about 1 kHz for gravitational antennas under construction. Variations in the metric of $h = 10^{-20}$ with interferometer base $L = 4.10^3$ m correspond to the sensitivity level $\Delta x = 2 \cdot 10^{-15}$ m.

The results of a series of works in which experiments on the study of the effect of various noises [10], among them seismic [5], are discussed, enable us to draw a conclusion on the reality of the indicated values of energy of low-frequency random oscillations of a test body.

Thus, the examined mechanism of the transformation of low-frequency oscillations of a test body to its high-frequency fluctuations can significantly limit the sensitivity of laser interference gravitational antennas. In this case, as it follows from (1) and (3), the proposed mechanism connected with radiation pressure on the test body leads to irregular high-frequency fluctuations of the intensity of light radiation I emergent from a Fabry—Perot resonator and, consequently, to increase in the high-frequency noise level of the gravitational antenna. In order to reduce the effect of this mechanism of high-frequency random phenomenon, it is necessary during design of these antennas to either reduce the laser light source intensity I_0 or increase the parameter $\varkappa$, by shifting the operating point of the resonator to a lower sensitivity region.

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