Physical principles of the design of an interferometer with a rotating plate

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This paper presents the design of an interferometer with a rotating plate for the wavelength range 8–12 µm. It is shown that the greatest difference of the optical path lengths in the interferometer arms is reached when the plate's refractive index lies in the range 1.3–1.4. For a plate 10 mm thick, the maximum achievable spectral resolution is predicted to be 0.8 cm^{-1}. The optimum range of angles of rotation of the plate is found for which the angular dependence of the optical path-length difference is linear. The technical appearance of an interferometer with no reference laser channel is discussed.

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INTRODUCTION

Fourier spectrometers of dynamic type, for all their ideological simplicity, possess a number of complex design elements. These can include a movable mirror assembly with strictly rectilinear motion of the latter and orientational accuracy of the mirrors in space to within a few arc seconds. It is not a trivial task to create a system for controlling a linear motor. The design is not simplified by using a reference channel (separate or combined with the main interferometer and a single-mode He/Ne laser as a reference source.[1] Such indispensable elements characterize the implementation of classical layouts of a Michelson interferometer, but a design exists that replaces the linear motion of the mirrors with oscillatory motion by means of a pendulum mirror suspension—a retroreflector. It is this interferometer design that is most widely used today in earth-based mobile Fourier-spectroradiometer complexes and in space-based apparatus.[1–4]

In recent years, D&P Instruments (USA)[1)] have developed and put into series production a Fourier spectroradiometer the uses an optical layout with a rotating plane-parallel plate (Fig. 1). The results attract attention to this spectrometer[5,6] because of its high scanning frequency and adequate spectral resolution. Its main advantage is the technical simplicity of the scanning system. However, whereas, before, the physical characteristics of the optical layouts under consideration, based on a Michelson interferometer, were usually essentially mathematically transparent, the following questions arise when an interferometer with a rotating plate is being developed:

1. What plate material is most effective in the given optical layout—with a large or a small refractive index?
2. How does the optical path difference of the beams in the interferometer depend on the angle of rotation of the plate and its thickness?
3. What are the requirements on the parameters of the interferometer as a function of the characteristics of the analog-to-digital converters (ADCs), the photodetectors, and the working range of the spectrum?

The goal of this article is to answer these questions.

THE INTERFEROMETER DESIGN

It is possible in principle to record four identical two-sided interferograms in a complete rotation of the plate around its axis, provided that the beams in the interferometer cross each other at 90° angles. Figure 2 shows part of an optical layout with beams that cross at 90°. To make the calculation simple and obvious, we assume that the optical axes of the interferometer beams cross at the surface of the rotating plate. Obviously, when the angles of rotation are $\alpha = 0°, 90°, 180°$, and 270°, the optical-path difference is zero, and this corresponds to the zero point of the interferogram. For convenience of the interferometer design, we assume the following constants: The thickness of the plate is $d = 10$ mm, and the angle of incidence of the rays is 45°. The optical path difference of the beams is then

$$\Delta L = 2(nOB + BC - nOA), \tag{1}$$

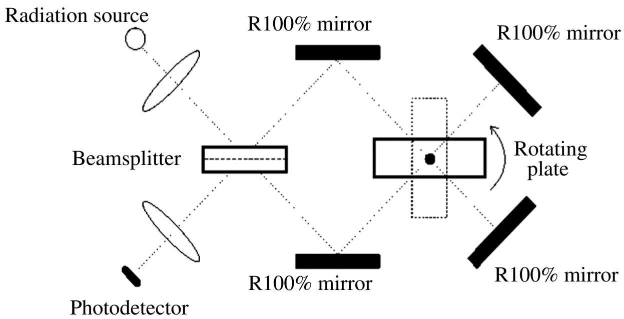

FIG. 1. Fundamental layout of a Fourier spectrometer with a rotating plane-parallel plate.

where n is the refractive index of the plate material, and OA, OB, and BC are the lengths of the corresponding line segments. It is easy to determine from elementary geometrical considerations, the sine theorem, and the refraction law of light that

$$OA = d/\sin(\gamma); \quad \sin(\gamma) = \sin(\delta + \alpha)/n; \quad \gamma = \arcsin(\sin(\delta + \alpha)/n). \tag{2}$$

$$OB = d/\cos(\beta); \quad \sin(\beta) = \sin(\delta - \alpha)/n; \quad \beta = \arcsin(\sin(\delta - \alpha)/n). \tag{3}$$

$$BC = d(\tan(\gamma) + \tan(\beta))\sin(\alpha)/\sin(\angle BCA); \quad \angle BCA = (\pi/2) - \delta. \tag{4}$$

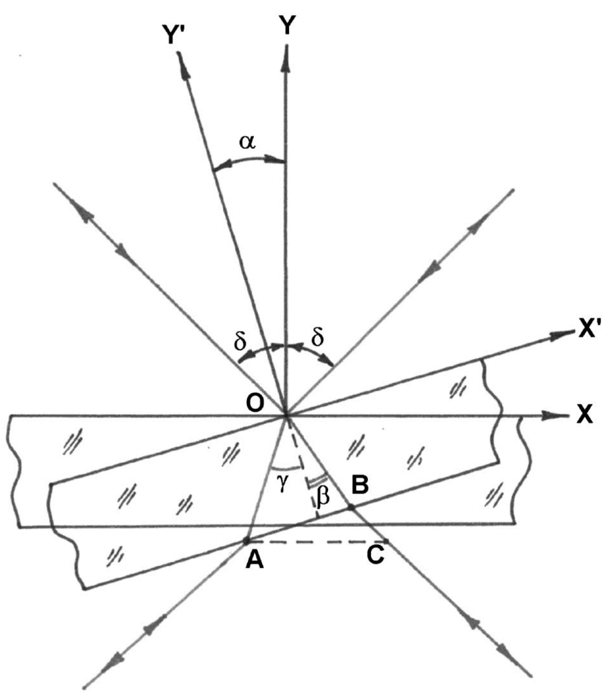

FIG. 2. Optical diagram of the ray path in the interferometer plate.

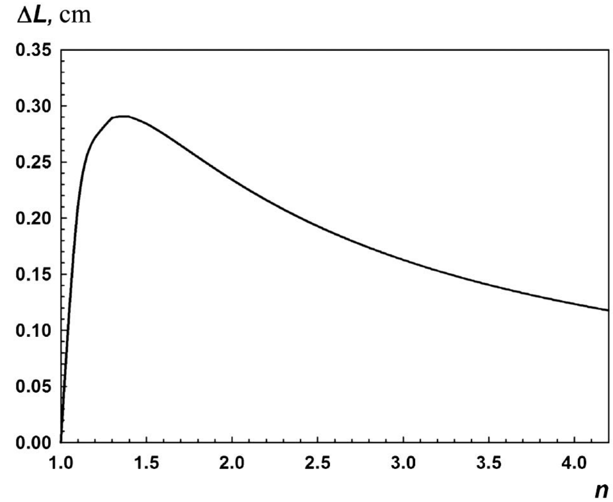

FIG. 3. Optical path difference in the interferometer vs the refractive index, with the rotation angle of the plate fixed at 15°.

The results of calculations from these relationships are plotted in Fig. 3 for a fixed angle of rotation of the plate $\alpha = 15°$ and in Fig. 4 for the most widely used materials of IR optics (KCl, KBr, ZnSe, and Ge) when working in the range 8–12 μm, as a function of the angle of rotation α. As shown by the calculations, the maximum possible value of ΔL is reached in materials with $n \approx 1.3-1.4$. The most suitable material for the range 8–12 μm is potassium bromide, KBr, although it is hygroscopic. It follows from these relationships that the optical path difference is directly proportional to the thickness d of the plate. For 10-mm-thick plates made from KCl or KBr, the maximum optical path difference reaches values of 0.7–0.8 cm, which corresponds to a limiting spectral resolution $\delta\nu$ around $1.25-1.4$ cm^{-1}.

The calculated dependences of ΔL on the angle of rotation of the plate are extremely close to linear on a limited interval of angles, as is clearly illustrated in Fig. 4. Such a calculated

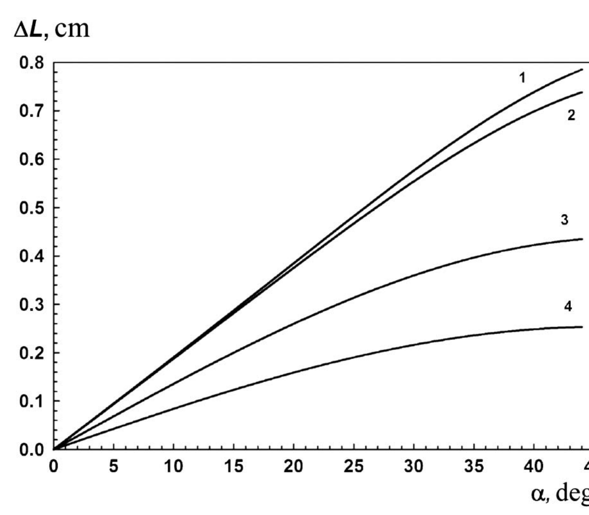

FIG. 4. Results of calculating the optical path difference for $\lambda = 10$ μm in the interferometer for various optical materials as a function of the angle of rotation of the plate: 1. KCl ($n = 1.4566$), 2. KBr ($n = 1.5265$), 3. ZnSe ($n = 2.4006$), 4. Ge ($n = 4.0040$).

dependence for KBr can be approximated by a straight line for angles of rotation of the plate within the limits of −30° to +30°. The absolute accuracy of the linear approximation is ≈60 μm for the indicated range. When the angular range is reduced to −20°, …, +20°, the accuracy of the linear approximation improves by more than an order of magnitude, and the absolute linearization error is less than 2 μm (Fig. 5); this is better than $\lambda/4$, with a relative error of 0.056% and a correlation coefficient of $R = 0.9999998$. The linear approximation equation is written in the form $\Delta L = (A + B\alpha)$, where $A = 1.0831444 \times 10^{-17}$ cm and $B = 0.0187896$ cm/deg. With such a limitation of the range of working angles and a plate thickness of 10 mm, the spectral resolution is just better than 3 cm^{-1}. To achieve a spectral resolution of 4 cm^{-1}, the required angular sweep is from −15° to +15°. We should emphasize that the constant term A in the linear-approximation equation can be neglected with no loss of accuracy, because it is small. The equation is transformed in this case into $\Delta L = B\alpha$. By having with suitable accuracy a linear dependence of the optical path difference on the angle of rotation, it is possible in principle to do without a laser reference channel, provided that one uses an angle encoder[2]—absolute or incremental. It should be pointed out here that temperature affects the optical path difference in this interferometer. Thus, for KBr in the range ±60°C, the temperature coefficients of the refractive indices are the negative quantity $n_T = (-3.95, \ldots, -4.29) \times 10^{-5}\,{}^\circ\mathrm{C}^{-1}$, and the linear resolution is the positive quantity $\alpha_T = (3.66, \ldots, 3.96) \times 10^{-5}\,{}^\circ\mathrm{C}^{-1}$. This means that, as the temperature of the plate varies, the temperature coefficients "play" on one side; i.e., the optical path difference of the rays in the interferometer increases as the temperature increases, and vice versa. This circumstance must be taken into account in operation if the temperature falloff is large, while no laser reference channel is included in the design.

However, it should be noted here that, for a large working range of angles, the Fresnel losses to reflection increase sharply for rays close to grazing incidence, and this in the final analysis produces instrumental apodization and, as a

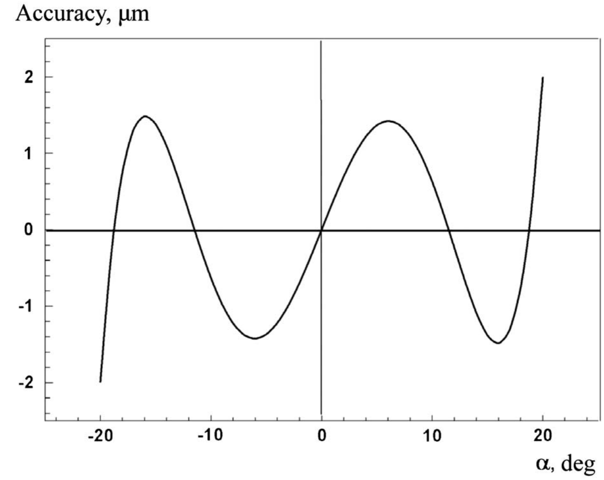

FIG. 5. Absolute accuracy of approximating the calculated curve by a linear function vs the angle of rotation of the plate.

consequence, reduces the limiting spectral resolution of the system. Thus the reflectance of the light beam from only one face of the plate reaches 60% when natural light is incident on a KBr plate at an angle of 85° to the normal ($\alpha = 40°$, Fig. 2), whereas the reflectance is less than 4% for normal incidence of the rays.

Next let the plate rotate with angular velocity n_0 rev/sec and let the working range of recording angles of the interferogram be from $-\varphi$ to $+\varphi$ degrees, and then the time to scan one two-sided interferogram is

$$\tau = \Delta\varphi/360 n_0 = \varphi/180 n_0. \quad (5)$$

The frequency range of the photodetector in this case must be no narrower than

$$f_{\min} = 2\Delta L/(\tau \nu_{\min}); \qquad f_{\max} = 2\Delta L/(\tau \nu_{\max}), \quad (6)$$

where $(\nu_{\min}, \nu_{\max})$ is the working range of the wave numbers of the spectrometer. The number of scans in one rotation of the plate equals four. For definiteness, let us have a KBr plate 10 mm thick that rotates in the interferometer at a rate of 300 rpm. This corresponds to a scanning frequency of 20 sec^{-1}. We specify the working range of wavelengths that characterizes the Fourier spectroradiometers as 8–12 μm (800–1250 cm^{-1}). The working range of angles −15°, …, +15° unambiguously determines the optical path difference $\Delta L = 0.25$ cm and the spectral resolution $\delta\nu = (1/\Delta L) = 4\ \mathrm{cm}^{-1}$. Substituting the starting data into Eqs. (5) and (6), we get

1. The scan time of one two-sided interferogram is $\tau \approx 0.017$ sec.
2. The frequency range of the photodetector is $f_{\min} =$ 24 kHz to $f_{\max} = 39$ kHz. These figures indicate that it is inexpedient to use pyroelectric and microbolometric photodetectors because their frequency range is limited, and virtually the only candidate is a photodetector based on the compound mercury–cadmium–tellurium, whose upper limit of the frequency range for photoresistance exceeds 100 kHz.
3. The minimum necessary number of points for a two-sided interferogram theoretically equals $N = 2(2\Delta\nu)/\delta\nu = 450$. The fast Fourier transformation imposes a limitation on the number of points in the working array, whereby their number in the interferogram must equal $N = 2^m = 512$, where m is a whole number. The analog-to-digital converter must possess signal-frequency discretization no less than $3f_{\max} = 120$ kHz.[3]
4. The divergence angle of the pencil of light rays in the interferometer is $\Omega = (2\pi\delta\nu)/\nu_{\max} = 0.02$ sr, and this corresponds to a plane flare angle of the cone of about 8°.

All of the overall sizes of the mirrors and of the beam-splitter system are easy to calculate, and the calculation process presents no special difficulties. However, the individual design modules—in particular, the module for attaching and rotating the plate—raises technical and technological questions. Thus, a DMS375 or DSO32 reducerless, low-speed, synchronous, multipole electric motor (designed with a direct drive shaft to the plate) can be used in this module. Unfortunately, the literature contains no parameters concerning the rotational stability (the flutter factor) of such motors as the load varies. If the rotational frequency of the latter were a constant quantity, because of the linear dependence of ΔL on the angle of rotation, it would be logical to use a separate, frequency-stabilized quartz generator in this interferometer design to trigger the ADC.

Variation of the rotational velocity during the scan causes false lines with amplitude $\pi k/(2\tau)$ to form in the spectrum, where $k = \Delta n_0/n_0$ is the flutter factor.[4] However, it is unlikely that mechanical perturbations at such low rpm would correspond to a frequency range of tens of kHz and would exert a negative noise effect on the recorded interferogram. This assertion can be checked only experimentally; nevertheless, estimating the flutter factor for perturbations with a "white-noise" spectrum gives the expression $k \leq 5 \times 10^{-3} N^{1/2}$,[7] and for $N = 450$ has a value of about 10%. Slow deviation of the rotational frequency, caused by changes of temperature, of the slope angles of the motor axle, and of the quality of the supply voltage, results in an error of the spacing of the digitization points on the interferogram, and, as a consequence, causes an error of the wavelength restoration in the spectrum. It is possible to roughly estimate the flutter factor (for a rotational rate different from the nominal value but constant in a given scan), which for a spectral resolution of 4 cm^{-1} and a working width of the spectrum of 450 cm^{-1}, must be less than

$$k < 100\Delta n_0/n_0 = 100\delta\nu/((2, \ldots, 4)\Delta\nu) = 0.2, \ldots, 0.4\%. \quad (7)$$

Considering that the flutter factor can reach values of a few percent as the load on the electric motor varies, it becomes crucial to solve the problem of long-term stability of the rotational frequency. If the condition given in Eq. (7) is satisfied, a simple technical solution suggests itself—to use an encoder to trigger the ADC at specific angular limits of rotation of the plate. The same encoder can be used as a feedback sensor in a system for electronic stabilization of the rotational frequency of the plate. Such stabilization systems, based on comparing the frequencies of the encoder pulses with a reference generator, were used earlier in high-class sound-reproduction apparatus (tape recorders, vinyl phonograph records) and provided a flutter factor at the 0.1% level or less.

It is obvious that the use of IR optics made from KBr requires a sealed box with an air dehumidifier to be used for the interferometer. As shown by an estimate, the requirements on the processing quality of the rotating plate lie within generally accepted technological norms. No antireflection coatings need to be deposited, taking into account the low refractive index of KBr–although, on the other hand, their deposition only improves the protection of the potassium bromide crystals from the possible action of water vapor if, of course, the coating itself is moisture-proof.

Here only one version of technical solution has been considered—the use of a plane-parallel plate. It is possible to use cubes in the design. A simple analysis shows that they make it possible to increase the scanning frequency by a factor of 2, but their use appreciably increases the rotating mass.

We have so far been speaking of the mid-IR range of the spectrum (8–12 μm); however, there is no fundamental rule against creating a Fourier spectrometer for another spectral range, using the appropriate photodetectors and optical elements. There are no physical limitations on which way the plate rotates in the interferometer, provided the electronic stabilization system is not sensitive to its sign.

CONCLUSIONS

The optical layout considered here is promising for creating spectroradiometric complexes with medium spectral class of resolution. The transition from the linear displacements of the movable mirror that characterize a Michelson interferometer to rotation of a plate makes it possible to reduce the mass and size of the interferometer unit and its control system and to increase the response rate.

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[1)]http://www.dpinstruments.com/

[2)]A type LIR-238 incremental angular photoelectric displacement converter. Sensor size 38 × 26 mm. Hollow shaft. Resolving power optionally from 100 to 720 000 steps per turn. Method of transmitting data: pulses of TTL level. Limiting rotational frequency 10 000. Supply voltage ±5 V, required current up to 120 mA (data from site http://www.skbis.ru/).

[3)]Modern, low-cost 14-bit ADCs possess a signal-digitization frequency of 400 kHz or more.

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